Radiative $\pi\rho$ and $\pi\omega$ transition form factors in a light-front constituent quark model

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Abstract

The form factors of the $\pi\rho$ and $\pi\omega$ radiative transitions are evaluated within a light-front constituent quark model, using for the first time the eigenfunctions of a light-front mass operator reproducing the meson mass spectrum and including phenomenological Dirac and Pauli quark form factors in the one-body electromagnetic current operator. The sensitivity of the transition form factors both to the meson wave functions and to the constituent quark form factors is illustrated. It is shown that the measurement of the $\pi\rho$ and $\pi\omega$ radiative transitions could help in discriminating among various models of the meson structure.

The investigation of the radiative transition from the pion to a vector meson ($\pi\gamma^* \to V$) could be of great relevance for studying the meson structure and the mechanism of quark confinement. Experimental information on $\pi\gamma^* \to V$ transition could be provided by the extraction of the pion-in-flight contribution from the electroproduction cross section of vector mesons off the nucleon or light nuclei (cf. experiments planned at high-intensity electron accelerator facilities, like CEBAF [1]). As far as the theoretical side is concerned, it should be pointed out that for values of the squared four-momentum transfer $Q^2 \sim$ few $(GeV/c)^2$ (i.e., in the range of values of Q^2 accessible to present $CEBAF$ energies) hadron electromagnetic properties are expected to be affected by the non-perturbative aspects of the QCD description of exclusive processes (see, e.g., [2]-[4]). Therefore, while waiting for a complete derivation of hadron form factors from the fundamental theory, it is of interest to analyze exclusive processes, like the radiative transition form factors of mesons, adopting a relativistic constituent quark (CQ) model, in which the meson is described as a bound state of a constituent $q\bar{q}$ pair with all the other degrees of freedom being frozen in the effective CQ structure and effective $q\bar{q}$ interaction. For an extensive discussion about i) the phenomenological success of describing the pion as the hyperfine partner of the ρ meson and ii) the possibility to reconcile the CQ model with the complexity of the QCD, see Refs. [5] and [6], respectively.

The aim of this letter is to investigate the radiative transitions $\pi^+\gamma^* \to \rho^+$ and $\pi^0\gamma^* \to \omega$ for values of Q^2 up to few $(GeV/c)^2$ within a light-front CQ model. We make use of the meson wave functions already adopted in [7]-[8] for the investigation of the elastic π- and ρ-meson form factors, i.e. we adopt the eigenfunctions of a light-front mass operator, constructed from the effective $q\bar{q}$ Hamiltonian of Ref. [9] which reproduces the meson mass spectrum. In this letter, the elastic pion and transition $\pi\rho$ ($\pi\omega$) form factors are calculated, adopting an effective one-body electromagnetic (e.m.) current operator which includes both Dirac and Pauli form factors for the CQ's (note that in Ref. [7] the pion form factor has been investigated including the CQ Dirac form factor only). It is shown that the calculated meson form factors are sensitive to the high-momentum tail, generated in the wave function by the one-gluon-exchange (OGE) part of the effective $q\bar{q}$ interaction, as well as to the e.m. structure of the CQ's. In particular, when the effects of the configuration mixing in the meson wave functions are considered, a non-vanishing value of the CQ anomalous magnetic moment is required in order to reproduce the experimental values of the radiative decay widths of ρ and ω mesons; moreover, the comparison between our calculations and the experimental data on pion form factor yields information on the possible size of light CQ's. Finally, our predictions are compared with those obtained within various relativistic approaches, showing that the measurement of the $\pi\rho$ and $\pi\omega$ radiative transitions could help in discriminating among different models of the meson structure.

Let us remind that the matrix elements of the e.m. current operator $\hat{I}_\mu$ for the transition $\pi\gamma^* \to \rho$ (ω) can be written as

$$\langle P_\pi, 00|\hat{I}_\mu|P_{\rho(\omega)}, 1\lambda_{\rho(\omega)}\rangle = F_{\pi\rho(\omega)}(Q^2)\epsilon_{\mu\nu\alpha\beta}e^\nu(\lambda_{\rho(\omega)})P_\pi^\alpha P_{\rho(\omega)}^\beta \tag{1}$$

where $Q^2 = -q \cdot q$ is the squared four-momentum transfer, $F_{\pi\rho(\omega)}(Q^2)$ is the transition form

factor, P_π and $P_{\rho(\omega)}$ are the four-momenta of the pion and the ρ (ω) mesons, respectively, and $\lambda_{\rho(\omega)}$ identifies the helicity state of the ρ (ω) meson with polarization four-vector $e(\lambda_{\rho(\omega)})$. In this letter, an effective one-body e.m. current operator $\hat{I}_\mu$ is considered, viz.

$$\hat{I}_\mu = \sum_{j=q,\bar{q}} \left[F_1^{(j)}(Q^2)\gamma_\mu + F_2^{(j)}(Q^2) i\sigma_{\mu\nu} \frac{q^\nu}{2m_j} \right] \tag{2}$$

where $F_1^{(q)}(Q^2)$ and $F_2^{(q)}(Q^2)$ are the Dirac and Pauli quark form factors, normalized as $F_1^{(q)}(0) = e_q$ and $F_2^{(q)}(0) = \kappa_q$, with e_q and κ_q being the CQ charge and anomalous magnetic moment, respectively. It should be pointed out that the two-body currents necessary for fulfilling both the gauge and rotational invariances [10] have not been taken into account in our calculations, where phenomenological CQ form factors have been adopted.

As in [7, 8], the Poincaré-covariant state vectors $|P_\pi, 00\rangle$ and$|P_{\rho(\omega)}, 1\lambda_{\rho(\omega)}\rangle$ are constructed using the Hamiltonian light-front formalism [10]. Let us briefly remind that the intrinsic light-front kinematical variables are $\vec{k}_\perp = \vec{p}_{q\perp} - \xi \vec{P}_\perp$ and $\xi = p_q^+/P^+$, where the subscript $\perp$ indicates the projection perpendicular to the spin quantization axis, defined by the vector $\hat{n} = (0, 0, 1)$, and the *plus* component of a 4-vector $p \equiv (p^0, \vec{p})$ is given by $p^+ = p^0 + \hat{n} \cdot \vec{p}$; eventually, $\tilde{P} \equiv (P^+, \vec{P}_\perp) = \tilde{p}_q + \tilde{p}_{\bar{q}}$ is the total light-front momentum of the meson. In what follows, only the $^3S_1$ channel of the ρ (ω) meson is considered, being its D-wave component extremely small ($p_D \simeq 0.16\%$) (see [8]). Omitting for the sake of simplicity the colour degrees of freedom, the requirement of Poincaré covariance for the intrinsic wave function $\chi_\mu^j(\xi, \vec{k}_\perp, \nu\bar{\nu})$ of a meson with angular momentum j and helicity μ implies

$$\chi_\mu^j(\xi, \vec{k}_\perp, \nu\bar{\nu}) = \sqrt{\frac{M_0}{16\pi\xi(1-\xi)}}\ R_\mu^j(\xi, \vec{k}_\perp, \nu\bar{\nu})\ w^{q\bar{q}}(k^2) \tag{3}$$

where $\nu, \bar{\nu}$ are the quark spin variables, $k^2 \equiv k_\perp^2 + k_n^2$, $k_n \equiv (\xi - 1/2)M_0$, $M_0^2 = (m_q^2 + k_\perp^2)/\xi$ $+\ (m_{\bar{q}}^2 + k_\perp^2)/(1-\xi)$ and the factor R_μ^j arises from the Melosh rotations of the quark spins (see Ref. [11] for its explicit expression). The wave function $\chi_\mu^j(\xi, \vec{k}_\perp, \nu\bar{\nu})$ is the eigenvector of a mass operator $\mathcal{M} = M_0 + \mathcal{V}$, where the free mass operator M_0 acquires a familiar form: $M_0 = \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2}$, and $\mathcal{V}$ represents a Poincaré invariant interaction. Therefore, the radial wave function $w^{q\bar{q}}(k^2)$ appearing in Eq. (3) is an eigenfunction of the mass operator $M = M_0 + V$, obtained from the Melosh rotation of $\mathcal{M}$ [7, 8]. In particular M_0 commutes with the Melosh rotation, while V is the Melosh-rotated interaction $\mathcal{V}$, and it is i) independent upon the total momentum and the centre of mass coordinates, ii) rotationally invariant. Thus we have chosen $w^{q\bar{q}}(k^2)$ as the eigenfunction of the effective $q\bar{q}$ Hamiltonian, introduced by Godfrey and Isgur (GI) [9] for reproducing the meson mass spectra, viz.

$$H_{q\bar{q}}\ w^{q\bar{q}}(k^2)|j\mu\rangle \ \equiv\ \left[\sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} + V_{q\bar{q}}\right]\ w^{q\bar{q}}(k^2)|j\mu\rangle = M_{q\bar{q}} w^{q\bar{q}}(k^2)|j\mu\rangle \tag{4}$$

where m_q ($m_{\bar{q}}$) is the constituent quark (antiquark) mass, $M_{q\bar{q}}$ the mass of the meson, $|j\mu\rangle = \sum_{\nu\bar{\nu}}\ \langle\frac{1}{2}\nu\frac{1}{2}\bar{\nu}|j\mu\rangle|\frac{1}{2}\nu\rangle|\frac{1}{2}\bar{\nu}\rangle$ the equal-time quark-spin wave function and $V_{q\bar{q}}$ the effective $q\bar{q}$

potential, composed by a OGE term (dominant at short separations) and a linear-confining term (dominant at large separations). Three different forms of $w^{q\bar{q}}(k^2)$ will be considered, namely $w^{q\bar{q}}_{(conf)}$, $w^{q\bar{q}}_{(si)}$ and $w^{q\bar{q}}_{(GI)}$ corresponding to the solution of Eq. (4) obtained using for $V_{q\bar{q}}$ only the linear confining term, the spin-independent part and the full GI interaction, respectively. It should be pointed out that: i) in case of both $w^{q\bar{q}}_{(conf)}$ and $w^{q\bar{q}}_{(si)}$ the meson mass spectrum is badly reproduced, and ii) the π-meson ($^1S_0$ channel) and ρ-meson ($^3S_1$ channel) radial wave functions differ only when the spin-spin component of the $q\bar{q}$ interaction is considered; this means that: $w^{\pi}_{(conf)} = w^{\rho}_{(conf)} \equiv w_{(conf)}$, $w^{\pi}_{(si)} = w^{\rho}_{(si)} \equiv w_{(si)}$ and $w^{\pi}_{(GI)} \neq w^{\rho}_{(GI)}$. The four wave functions $w_{(conf)}$, $w_{(si)}$, $w^{\pi}_{(GI)}$ and $w^{\rho}_{(GI)}$ have been already reported in [8], where it has been shown that both the central and the spin-dependent components of the OGE interaction strongly affect the high-momentum tail of the π- and ρ-meson wave functions. Herebelow, an ideal mixing in the vector sector is assumed, because the effects of $\phi - \omega$ mixing are expected to affect slightly the $\pi^0\omega$ transition form factor (cf. Ref. [11]). For the same reason, also the effects of the $\rho^0 - \omega$ mixing are neglected. Therefore, the same radial wave function for the ρ and ω mesons is adopted. According to Ref. [9], the value $m \equiv m_q = m_{\bar{q}} = 0.220\ GeV$ is adopted.

Within the light-front formalism, the invariant hadron form factors can be determined using only the matrix elements of the component I^+ of the current operator; for space-like values of the four-momentum transfer, we choose a frame where $q^+ = 0$, for such a choice allows to suppress the contribution of the Z-graph (pair creation from the vacuum) [12]. Using Eqs. (1-3) it can be checked that the matrix element corresponding to $\lambda_{\rho(\omega)} = 0$ is vanishing; thus, considering $\lambda_{\rho(\omega)} = 1$ and performing a straightforward spin and flavour algebra [13], one gets

$$F_{\pi^+\rho^+}(Q^2) = \frac{1}{3} F_1^{(S)}(Q^2)\ H_1^{\pi\rho}(Q^2) + \frac{1}{3} F_2^{(S)}(Q^2)\ H_2^{\pi\rho}(Q^2) \tag{5}$$

$$F_{\pi^0\omega}(Q^2) = F_1^{(V)}(Q^2)\ H_1^{\pi\rho}(Q^2) + F_2^{(V)}(Q^2)\ H_2^{\pi\rho}(Q^2) \tag{6}$$

where $F^{(S,V)}_{1(2)}$ are the isoscalar and isovector parts of the constituent u and d quark form factors, given by $F^{(S)}_\alpha(Q^2) \equiv 3[F^{(u)}_\alpha(Q^2) + F^{(d)}_\alpha(Q^2)]$ and $F^{(V)}_\alpha(Q^2) \equiv F^{(u)}_\alpha(Q^2) - F^{(d)}_\alpha(Q^2)$ (with $\alpha = 1, 2$ and $F^{(S)}_1(0) = F^{(V)}_1(0) = 1$) [1]. In Eqs. (5)-(6) $H_1^{\pi\rho}(Q^2)$ and $H_2^{\pi\rho}(Q^2)$ are body form factors, which depend on the motion of the CQ's inside the mesons and are given explicitely by

$$H_1^{\pi\rho}(Q^2) = \int d\vec{k}_\perp d\xi\ \frac{\sqrt{M_0 M_0'}}{4\xi(1-\xi)}\ \frac{w^\pi(k'^2) w^\rho(k^2)}{4\pi}\ \frac{m\lambda + 2k_y^2}{\lambda\ \sqrt{m^2 + k_\perp^2}\ \sqrt{m^2 + k'^2_\perp}}\ 2(1-\xi) \tag{7}$$

$$H_2^{\pi\rho}(Q^2) = \int d\vec{k}_\perp d\xi\ \frac{\sqrt{M_0 M_0'}}{4\xi(1-\xi)}\ \frac{w^\pi(k'^2) w^\rho(k^2)}{4\pi}\ \frac{\lambda(m^2 + \vec{k}_\perp \cdot \vec{k'}_\perp) - 2M_0(1-\xi)k_y^2}{m\lambda\ \sqrt{m^2 + k_\perp^2}\ \sqrt{m^2 + k'^2_\perp}} \tag{8}$$

where $\lambda \equiv 2m + M_0$, $\vec{k'}_\perp \equiv \vec{k}_\perp + (1-\xi)\vec{q}_\perp$ and the x axis is chosen in the direction of $\vec{q}_\perp$ ($Q^2 \equiv |\vec{q}_\perp|^2$). The expression (7) for $H_1^{\pi\rho}(Q^2)$ has been already derived in [14], whereas

[1] In the derivation of Eqs. (5)-(6) the relation $F^{(\bar{q})}_\alpha(Q^2) = -F^{(q)}_\alpha(Q^2)$ has been used.

$H_2^{\pi\rho}(Q^2)$ (Eq. (8)) is a new body form factor related to the presence of the Pauli quark form factor in the one-body e.m. current operator (2). If the u and d constituent quarks have the same e.m. structure, which means $F_\alpha^{(S)} = F_\alpha^{(V)}$, the $\pi\omega$ transition form factor is three times the one corresponding to the $\pi\rho$ transition at any values of Q^2 and for any choice of the radial wave function $w^{q\bar{q}}$. The body form factors $H_1^{\pi\rho}$ and $H_2^{\pi\rho}$, calculated using the three radial wave functions $w_{(conf)}^{q\bar{q}}$, $w_{(si)}^{q\bar{q}}$ and $w_{(GI)}^{q\bar{q}}$, are shown in Fig. 1. It can be seen that the effects of the spin-dependent part of the effective $q\bar{q}$ interaction are negligible, because the products $w_{(si)}^{(\pi)} \cdot w_{(si)}^{(\rho)}$ and $w_{(GI)}^{(\pi)} \cdot w_{(GI)}^{(\rho)}$ are very similar in a wide range of values of the internal momentum (cf. Fig. 1 in [8]); in particular, note that $H_2^{\pi\rho}$ is approximately two times larger than $H_1^{\pi\rho}$.

The values of the transition form factors at $Q^2 = 0$ (the so-called transition magnetic moments) have been experimentally determined from the radiative decay widths of the ρ and ω mesons, viz. $\mu_{\pi^+\rho^+}^{exp} = 0.741 \pm 0.038$ (c/GeV) and $\mu_{\pi^0\omega}^{exp} = 2.33 \pm 0.06$ (c/GeV) [15]. From Eqs. (5)-(6) one gets $\mu_{\pi^+\rho^+} = [H_1^{\pi\rho}(0) + \kappa_S H_2^{\pi\rho}(0)]/3$ and $\mu_{\pi^0\omega} = H_1^{\pi\rho}(0) + \kappa_V H_2^{\pi\rho}(0)$, where $\kappa_S \equiv 3(\kappa_u + \kappa_d)$ and $\kappa_V \equiv \kappa_u - \kappa_d$ are the isoscalar and isovector u and d quark anomalous magnetic moments. Assuming $\kappa_S = \kappa_V = 0$ (which implies $\mu_{\pi^0\omega} = 3\mu_{\pi^+\rho^+}$), our results, obtained using the radial wave functions $w_{(conf)}^{q\bar{q}}$, $w_{(si)}^{q\bar{q}}$ and $w_{(GI)}^{q\bar{q}}$, are as follows: $\mu_{\pi^+\rho^+} = 0.833, 0.637, 0.561$ (c/GeV) respectively. This means that: i) the value obtained using $w_{(conf)}^{q\bar{q}}$ is close to the one reported in [11], where a simple Gaussian wave function was adopted; ii) the effects of the configuration mixing, due to the OGE interaction, lead to a $\sim 25\%$ underestimation of the experimental data. Then, the values of κ_S and κ_V can be chosen in order to reproduce the experimental values of the transition magnetic moments; in such a way, using $w_{(GI)}^{q\bar{q}}$, one gets: $\kappa_S = 0.174 \pm 0.037$ and $\kappa_V = 0.208 \pm 0.019$, corresponding to $\kappa_u = 0.133 \pm 0.016$ and $\kappa_d = -0.075 \pm 0.016$. Note that, within the quoted uncertainties, $\kappa_V \simeq \kappa_S$ due to the fact that $\mu_{\pi^0\omega}^{exp} \simeq 3\, \mu_{\pi^+\rho^+}^{exp}$. Since non-vanishing quark anomalous magnetic moments are required, the elastic form factor of the pion has to be calculated including also the contributions arising from the Pauli quark form factor not considered in [7]. Starting from the relation $\langle P'_\pi, 00 | \hat{I}_\mu | P_\pi, 00 \rangle = F_\pi(Q^2)\ (P + P')_\mu$ and using Eqs. (2-3), one has

$$F_\pi(Q^2) = F_1^{(V)}(Q^2)\ H_1^\pi(Q^2)\ +\ F_2^{(V)}(Q^2)\ H_2^\pi(Q^2) \tag{9}$$

with

$$H_1^\pi(Q^2) = \int d\vec{k}_\perp d\xi\ \frac{\sqrt{M_0 M'_0}}{4\xi(1-\xi)}\ \frac{w^\pi(k'^2) w^\pi(k^2)}{4\pi} \frac{m^2 + \vec{k}_\perp \cdot \vec{k'}_\perp}{\sqrt{m^2 + k_\perp^2}\ \sqrt{m^2 + k'^2_\perp}} \tag{10}$$

$$H_2^\pi(Q^2) = -\frac{Q^2}{2}\ \int d\vec{k}_\perp d\xi\ \frac{\sqrt{M_0 M'_0}}{4\xi(1-\xi)}\ \frac{w^\pi(k'^2) w^\pi(k^2)}{4\pi} \frac{1-\xi}{\sqrt{m^2 + k_\perp^2}\ \sqrt{m^2 + k'^2_\perp}} \tag{11}$$

The expression (10) for $H_1^\pi(Q^2)$ has been obtained in several papers (cf., e.g., [7]), whereas we stress that $H_2^\pi(Q^2)$ (Eq. (11)) is a new body form factor related to the presence of the Pauli quark form factor in the one-body e.m. current operator (2). The body form factors H_1^π and H_2^π, calculated using the three radial wave functions $w_{(conf)}^{q\bar{q}}$, $w_{(si)}^{q\bar{q}}$ and $w_{(GI)}^{q\bar{q}}$, are

shown in Fig. 2. It can be seen that the effects of the configuration mixing lead to a sharp increase of the body form factors H_1^π and $|H_2^\pi|$; in particular, note that $|H_2^\pi|$ is substantially larger than H_1^π for $Q^2 > 1\ (GeV/c)^2$. From Eqs. (5)-(6) and (9) it turns out that both F_π and $F_{\pi^0\omega}$ depends upon the isovector combination $F_\alpha^{(V)}(Q^2)$ of u and d quark form factors, whereas $F_{\pi^+\rho^+}$ involves their isoscalar part $F_\alpha^{(S)}(Q^2)$.

We have evaluated the elastic pion form factor and the transition $\pi\rho$ and $\pi\omega$ form factors, including in the one-body e.m. current both Dirac and Pauli quark form factors parametrized through simple monopole and dipole behaviours, respectively, viz.

$$F_1^{(q)}(Q^2) = \frac{e_q}{1 + (r_1^q)^2 Q^2/6} \quad , \quad F_2^{(q)}(Q^2) = \frac{\kappa_q}{(1 + (r_2^q)^2 Q^2/12)^2} \tag{12}$$

The parameters appearing in Eq. (12) have been fixed as follows. From Eq. (9) the pion charge radius is given by $< r^2 >_\pi \equiv -6 \frac{dF_\pi}{dQ^2}(0) =< r_1^2 >_{body} + \kappa_V < r_2^2 >_{body} + (r_1^V)^2$, where $< r_\alpha^2 >_{body} \equiv -6 \frac{dH_\alpha^\pi}{dQ^2}(0)$ and $(r_1^V)^2 \equiv e_u (r_1^u)^2 - e_d (r_1^d)^2$. Using the value $\kappa_V = 0.208$, obtained from our previous analysis of the transition magnetic moments $\mu_{\pi^+\rho^+}$ and $\mu_{\pi^0\omega}$, and the experimental value $< r^2 >_\pi^{exp} = (0.660 \pm 0.024\ fm)^2$ [16], one gets $r_1^V = 0.41\ fm$. If the e.m. structure of u and d CQ's is assumed to be the same, one has $r_1^u = r_1^d = r_1^V = 0.41\ fm$. Finally, the value $r_2^u = r_2^d = 0.52\ fm$ has been chosen in order to get agreement in the whole range of existing pion data, as it is shown in Fig. 3a, where our results are reported and compared with the predictions of a simple Vector Meson Dominance (VMD) model (i.e., $F_\pi^{VMD}(Q^2) = 1/(1 + Q^2/M_\rho^2)$, with M_ρ being the ρ-meson mass), the results of the Bethe-Salpeter (BS) approach of Ref. [18] and those obtained in Ref. [3] using QCD sum rule techniques. It can be seen that the differences among the theoretical calculations are quite small, so that the existing pion data do not discriminate among various models of the meson structure. Using the same CQ form factors adopted for the description of the pion, we have calculated the transition form factor $F_{\pi\rho}(Q^2)$. Our results are reported in Fig. 3b, where they are compared with the predictions of the simple ρ-pole VMD model and the results of Refs. [19] (BS approach) and [4] (QCD sum rule technique). It can be seen that, unlike the case of the pion form factor, the differences among various relativistic calculations of the $\pi\rho$ transition form factor are quite sizeable at $Q^2 > 1\ (GeV/c)^2$; therefore, the measurement of $F_{\pi\rho}(Q^2)$ could help in discriminating among various models of the meson structure. It is worth mentioning that the $\pi\rho$ and $\pi\omega$ transition form factors play a relevant role in the contribution of meson exchange currents (MEC) to the deuteron elastic form factors (cf. Ref. [19]); in particular, the model dependence, clearly exhibited in Fig. 3(b) at large Q^2, could produce sizable differences in the MEC contribution to the deuteron form factors.

It should be pointed out that the contributions of the Pauli quark form factor to the elastic pion and transition $\pi\rho$ ($\pi\omega$) form factors turn out to be at most $\sim 30\%$. Then, from Eqs. (5)-(6) it follows that the ratio of the $\pi\omega$ to the $\pi\rho$ transition form factor depends slightly on the meson wave function $w^{(q\bar{q})}$ and is given mainly by the ratio of the isovector to the isoscalar Dirac quark form factors, viz. $F_{\pi\omega}(Q^2)/F_{\pi\rho}(Q^2) \sim 3\ F_1^{(V)}(Q^2)/F_1^{(S)}(Q^2)$. Therefore, the ratio of the $\pi\omega$ to the $\pi\rho$ transition form factor is expected to be sensitive to possible differences between $F_1^{(V)}(Q^2)$ and $F_1^{(S)}(Q^2)$, i.e. to possible differences in the e.m.

structure of the constituent u and d quarks. These features are illustrated in Fig. 4, where our results, obtained assuming various choices of the size parameters appearing in Eq. (12), are reported. It should be pointed out that each set of values adopted for the CQ size parameters satisfies the constraints $e_u(r_1^u)^2 - e_d(r_1^d)^2 = (0.41\ fm)^2$ and $e_u(r_2^u)^2 - e_d(r_2^d)^2 = (0.52\ fm)^2$, so that the results of the corresponding calculations of the elastic pion form factor remain in nice agreement with the existing pion data.

In conclusion, the radiative transition $\pi^+\gamma^* -> \rho^+$ and $\pi^0\gamma^* -> \omega$ have been analyzed within a constituent quark model, in which the relativistic treatment of the light constituents is achieved using the Hamiltonian light-front formalism. In our approach the eigenfunctions of a light-front mass operator, reproducing the meson mass spectrum, are adopted and both Dirac and Pauli form factors are included in the one-body electromagnetic current operator. When the effects of the configuration mixing, generated in the meson wave function by the one-gluon-exchange interaction, are considered, a non-vanishing value of the constituent quark anomalous magnetic moment is required in order to reproduce the experimental values of the radiative decay widths of ρ and ω mesons. The contributions resulting both from Dirac and Pauli quark form factor have been evaluated also in case of the pion and the comparison of our results with existing pion data has provided information on the size parameters of light constituent quarks. Moreover, it has been shown that the ratio of the $\pi\omega$ to the $\pi\rho$ transition form factor is not affected by the choice of the meson wave function, whereas it is sharply sensitive to possible differences in the electromagnetic structure of u and d constituent quarks. Our predictions have been compared with the results of various relativistic approaches, showing that the measurements of $\pi\rho$ and $\pi\omega$ radiative transitions could help in discriminating among different models of the meson structure.

Acknowledgements. We gratefully acknowledge E. Pace for useful discussions and F. Gross for supplying us with the numerical output of the calculations of Ref. [19].

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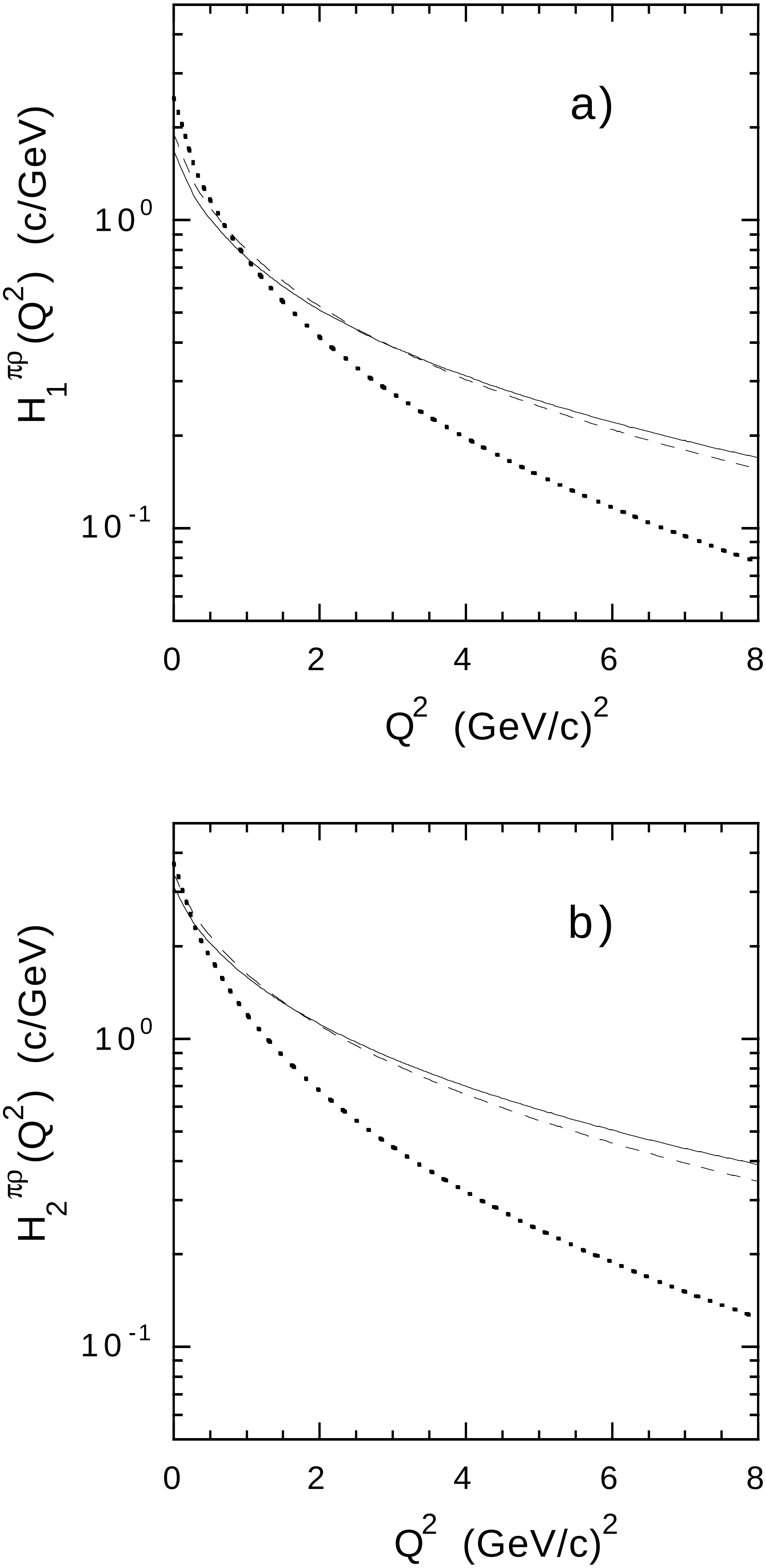
Figure Captions

Fig. 1. a) The body form factor $H_1^{\pi\rho}$ (Eq. (7)) vs. Q^2. The dotted, dashed and solid lines correspond to the calculations performed using the radial wave functions $w^{\pi}_{(conf)}$, $w^{\pi}_{(si)}$ and $w^{\pi}_{(GI)}$, which are the solutions of Eq. (4) obtained using for $V_{q\bar{q}}$ only the linear confining term, the spin-independent part and the full GI interaction [9], respectively. b) The same as in a), but for $H_2^{\pi\rho}$ (Eq. (8)).

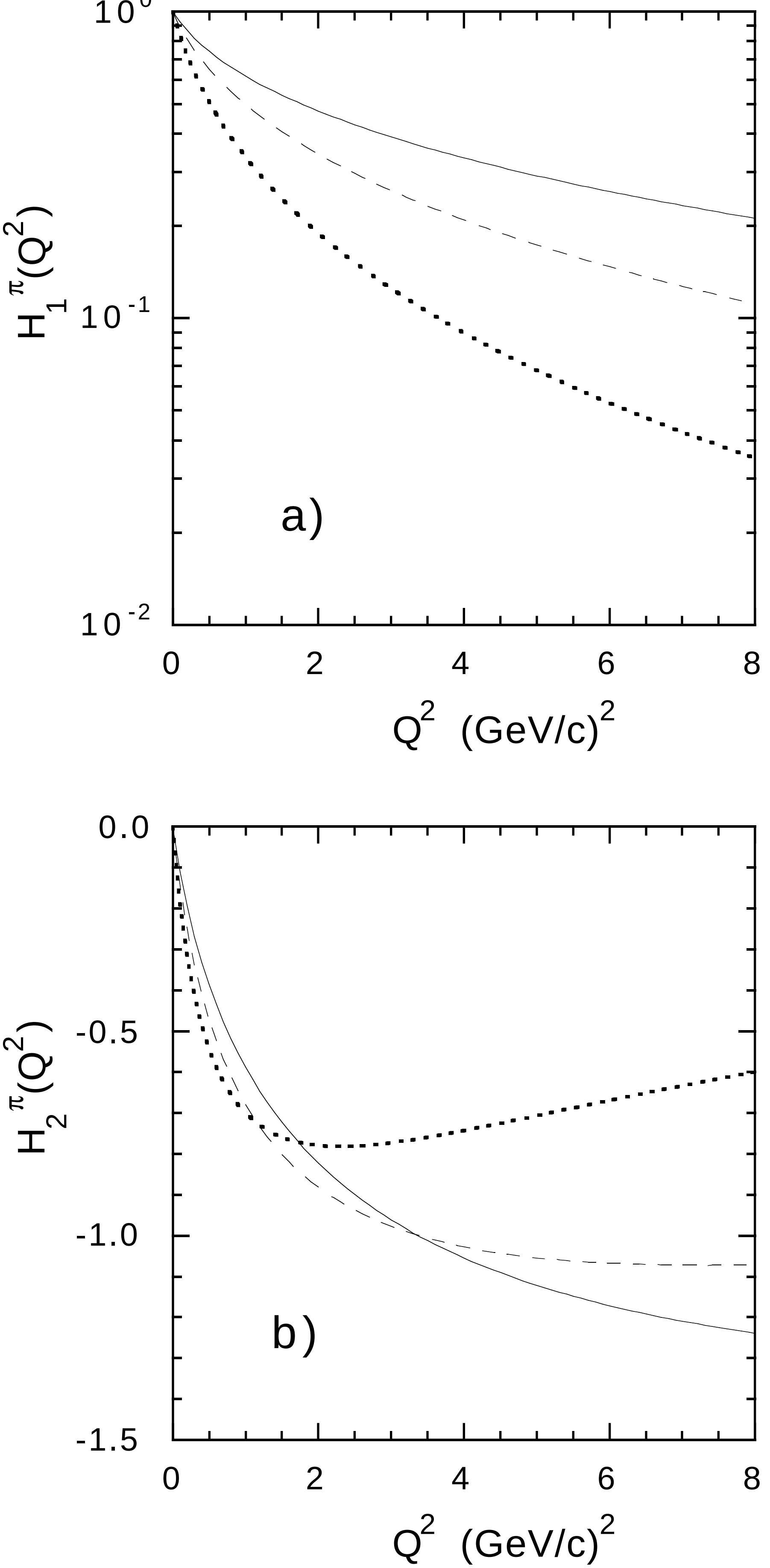
Fig. 2. a) The body form factor H_1^{π} (Eq. (10)) vs. Q^2. The dotted, dashed and solid lines correspond to the calculations performed using the radial wave functions $w^{q\bar{q}}_{(conf)}$, $w^{q\bar{q}}_{(si)}$ and $w^{q\bar{q}}_{(GI)}$, respectively. b) The same as in a), but for H_2^{π} (Eq. (11)).

Fig. 3. a) The elastic form factor of the pion (Eq. (9)), times Q^2, vs. Q^2. Our results, obtained using $w^{\pi}_{(GI)}$ and considering the CQ form factors of Eq. (12) (with $r_1^u = r_1^d = 0.41\ fm$, $r_2^u = r_2^d = 0.52\ fm$ and $\kappa_V = 0.208$), are represented by the solid line. For comparison, the experimental data of Refs. [16, 17] (cf. also [7]) are reported. The dotted, dot-dashed and dashed lines correspond to the predictions of a simple VMD model (ρ-meson pole only), the BS approach of Ref. [18] and the QCD sum rule technique of Ref. [3], respectively. b) The form factor of the radiative transition $\pi^+\gamma^* -> \rho^+$ (Eq. (5)), times Q^4, vs. Q^2. The solid line correspond to our results, obtained using $w^{q\bar{q}}_{(GI)}$ and considering the same constituent quark form factors as in (a). The dotted, dot-dashed and dashed lines correspond to the predictions of a simple VMD model (ρ-meson pole only), the BS approach of Ref. [19] and the QCD sum rule technique of Ref. [4], respectively.

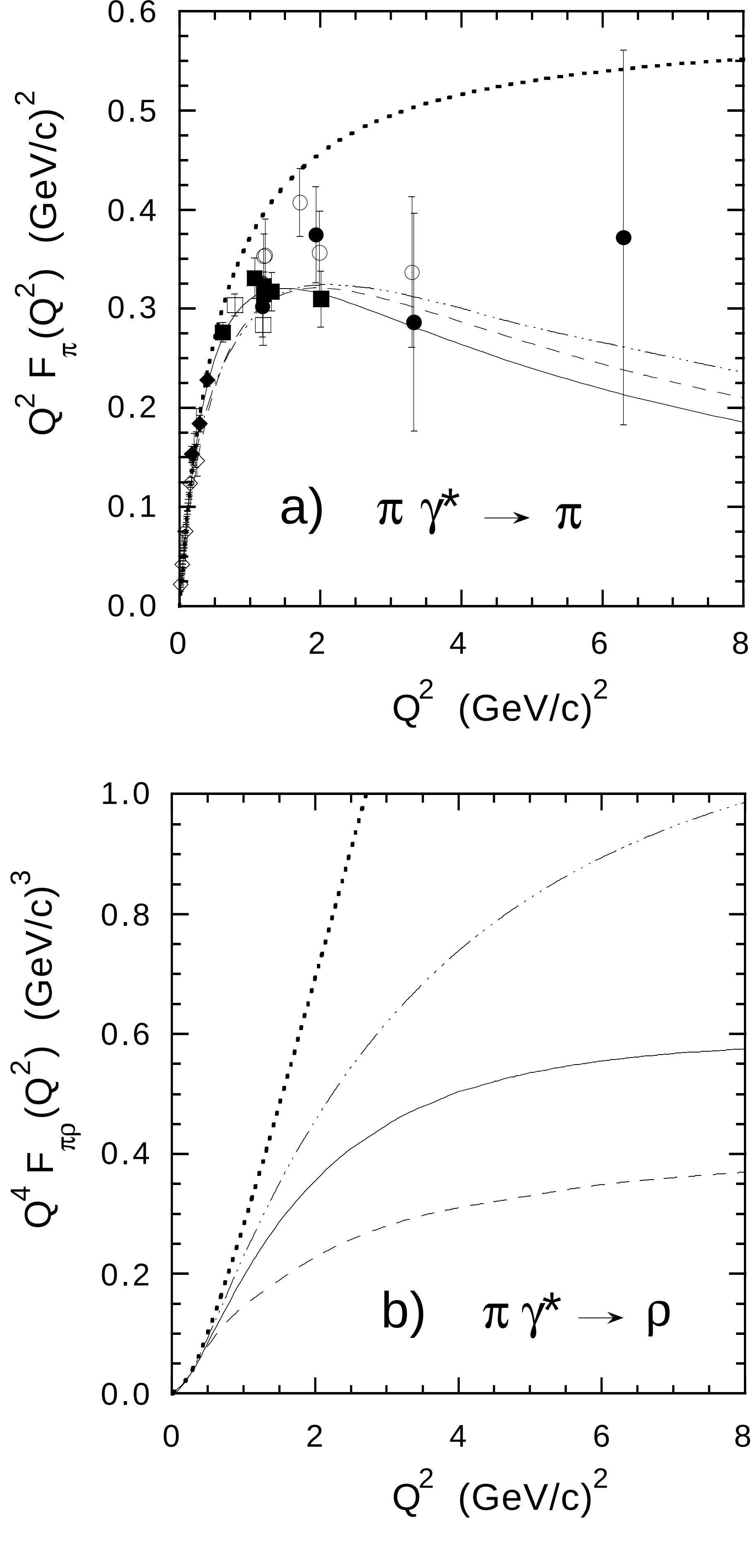
Fig. 4. The ratio of the $\pi^0\omega$ to the $\pi^+\rho^+$ radiative transition form factor (Eqs. (5)-(6)) vs. Q^2. The dotted, dashed and solid lines correspond to the calculations performed using $w^{q\bar{q}}_{(conf)}$, $w^{q\bar{q}}_{(si)}$ and $w^{q\bar{q}}_{(GI)}$, respectively, and assuming the same e.m. structure for the constituent u and d quarks (viz., $r_1^u = r_1^d = 0.41\ fm$ and $r_2^u = r_2^d = 0.52\ fm$). The solid lines with open and full dots are the results of the calculations obtained using $w^{q\bar{q}}_{(GI)}$ and assuming different sizes for the constituent u and d quarks; namely, the parameters appearing in Eq. (12) are: $r_1^u = 0.37\ fm < r_1^d = 0.49\ fm$, $r_2^u = 0.46\ fm < r_2^d = 0.61\ fm$ (open dots), and $r_1^u = 0.44\ fm > r_1^d = 0.34\ fm$, $r_2^u = 0.56\ fm > r_2^d = 0.43\ fm$ (full dots). In all cases $\kappa_V = 0.208$ and $\kappa_S = 0.174$ (see text). Note that, for each set of values of the CQ size parameters, the corresponding calculation of the elastic pion form factor is in nice agreement with existing pion data.



F. Cardarelli et al., Physics Letters B: fig. 1



F. Cardarelli at al., Physics Letters B: fig. 2



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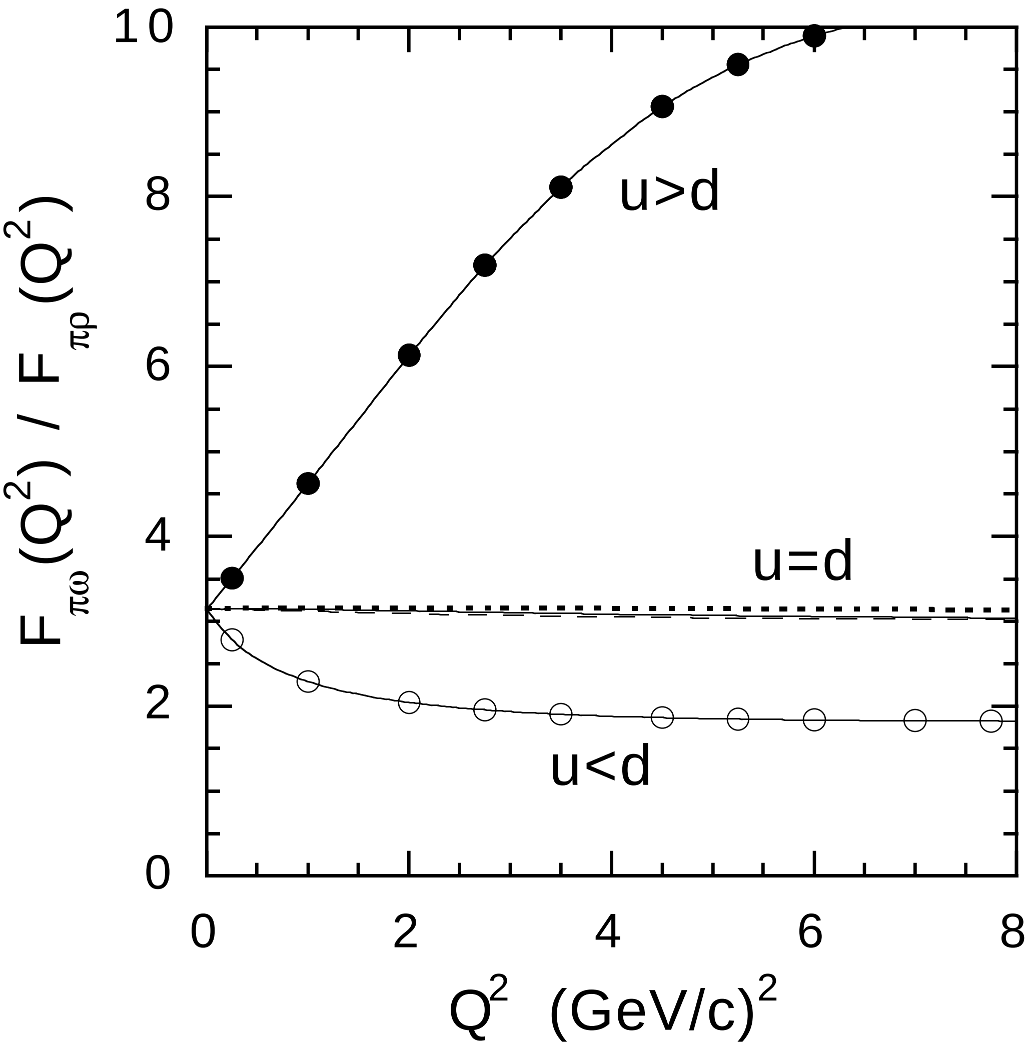

F. Cardarelli et al., Physics Letters B: fig. 4